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ABSTRACT

This study was designed to derive the distribution of a test statistic based on normal probability plots. The first purpose was to provide an empirical derivation of the critical values for the Line Test (LT) with an extensive computer simulation. The goal was to develop a test that is sensitive to a wide range of alternative distributions, applicable to a wide range of sample sizes, and easy to compute. The second purpose was to determine the power of LT and compare it to that of several other test statistics. Monte Carlo simulation was used to generate the critical values of LT by randomly generating 500,000 replications from a normal distribution for sample sizes 10(1)100(25)1000(250)5000. For each replication, the value of LT was calculated. The empirical critical values were determined for each of three levels of significance for each sample size. These critical values were then "smoothed" using nonlinear regression techniques. The results indicate that LT provides adequate control over Type I errors while at the same time providing statistical power comparable to the Shapiro-Wilk test. Results also indicate that LT is easy to compute, is powerful for detecting departures from normality under a wide variety of alternative distributions, and is available for sample sizes up to 5,000. An appendix contains the Statistical Package for the Social Sciences syntax for computing the LT for a given data set. (Contains 2 figures, 10 tables, and 45 references.) (Author/SLD)

A Monte Carlo Simulation of an Omnibus Test Based on Normal Probability Plots: The Line Test

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Abstract

Currently, there is disagreement regarding the importance of the assumption of normality; many statistical techniques are robust to violations of normality under some conditions, but such robustness is not universal. Furthermore, there is no generally accepted test for detecting departures from normality.

The current study was designed to derive the distribution of a test statistic based on normal probability plots. The first purpose was to provide an empirical derivation of the critical values for the Line Test (*LT*) with an extensive computer simulation. The goal was to develop a test that is sensitive to a wide range of alternative distributions, applicable to a wide range of sample sizes, and easy to compute. The second purpose was to determine the power of *LT* and compare it to that of several other test statistics.

Monte Carlo simulation was used to generate the critical values of *LT* by randomly generating 500,000 replications from a normal distribution for sample sizes 10(1)100(25)1000(250)5000. For each replication, the value of *LT* was calculated. The empirical critical values were determined for each of three levels of significance for each sample size. These critical values were then "smoothed" using nonlinear regression techniques.

The results indicated that *LT* provides adequate control over Type I errors while at the same time providing statistical power comparable to the Shapiro-Wilk test. In conclusion, *LT* is easy to compute, is powerful for detecting departures from normality under a wide variety of alternative distributions, and is available for sample sizes up to 5,000.

A Monte Carlo Simulation of an Omnibus Test Based on Normal Probability Plots: The Line Test

Introduction

The normal probability distribution is the most widely used distribution in statistics as the assumption of normality allows us to employ some very sophisticated analytic techniques in the interpretation of our data. However, improper analysis of data that deviate from normality can lead to inaccurate conclusions. Therefore, knowing how to determine whether a sample of measurements is from a normally distributed population is crucial both in the development of statistical theory and in practice. Currently, there appears to be some level of disagreement regarding the importance of the assumption of normality. It has been established that many statistical techniques are robust to violations of normality under some conditions, but such robustness is not universal (Tabachnick and Fidell, 1996). Among the non-robust procedures are tests on variances (Box, 1953), structural equation modeling (Bollen, 1989; Wang, Fan, and Willson, 1996), and meta-analysis (Glass, McGaw, and Smith, 1981; Greenhouse and Iyengar, 1994; Hedges and Olkin,1985).

Much effort has been exerted in developing techniques solely for the purpose of detecting departures from normality. This effort began as early as the late 19^{th} century with Pearson's (1895) work on moments, particularly the third and fourth moments which are commonly referred to as the skewness and kurtosis coefficients, respectively. Since that time, there has been considerable attention devoted to these moments and how they may be used to assess departures from normality (e.g., Bowman & Shenton, 1973; Bowman & Shenton, 1975; D'Agostino, Belanger, & D'Agostino, 1990; D'Agostino & Cureton, 1972; D'Agostino & Pearson, 1973; D'Agostino & Teitjen, 1973; Fisher, 1930; Fisher ,1973; Pearson, 1930; Pearson, 1931; Pearson, 1963; Pearson, D'Agostino, & Bowman, 1977; Williams, 1935). In another direction, there has been extensive work developing omnibus tests based on normal probability plots (e.g., Brown & Hettmansperger, 1996; Filliben, 1975; Hegazy & Green, 1975; LaBrecque, 1977; Looney & Gulledge, 1985; Royston, 1982; Royston, 1992; Shapiro & Francia, 1972; Shapiro & Wilk, 1965; Shapiro, Wilk, & Chen, 1968; Verrill & Johnson, 1988; Weisberg & Bingham, 1975).

In addition to the two types of tests (moments and probability plots), there are several other "miscellaneous" tests that have been developed. Among them are D'Agostino's D statistic (D'Agostino, 1971), the Kolmogorov-

Smirnov test (Lilliefors, 1967), the ratio of the range to the standard deviation (David, Hartley, & Pearson, 1954), and the ratio of the mean deviation to the standard deviation (Geary, 1935)

While many tests currently exist, there is no gold standard among them as there is no one test which is sensitive to a wide range of alternative distributions, applicable to a wide range of sample sizes, and easy to compute. In fact, the variability in normality tests used is evident by noting that even the major statistical packages such as SAS, SPSS, STATA, SYSTAT, and BMDP have implemented different normality tests (D'Agostino, Belanger, & D'Agostino, 1990; Hopkins & Weeks, 1990; Ware & Ferron, 1995).

Many would argue that the Shapiro-Wilk (1965) *W* test is the most sensitive test to a wide range of alternative distributions. In fact, *W* was the first test for normality that was able to detect departures due to either skewness or kurtosis, or both. However, because of the complexity of this test, no statistical package has implemented *W* for sample sizes larger than 50. Large sample approximations have been developed (e.g., Royston, 1982), but Althouse (1997) showed that this approximation may not provide adequate control of the Type I error rate.

At this time, there are two tests for normality that seem to stand out from the others. D'Agostino and his colleagues have developed K^2, which is one of several recommended by Bollen (1989) and currently available within STATA. K^2 is defined as the sum of the standardized and normalized measures of skewness and kurtosis, and is compared to $\chi^2(2)$. The Shapiro-Wilk *W* test is defined by considering the regression of the ordered observed values on the expected deviates from a normal distribution and is recognized as the best of the test statistics based on normal probability plots. However, the computation of *W* requires a set of coefficients, which have been derived only for sample sizes less than or equal to 50 and approximated for larger sample sizes. Therefore, *W* is usually limited to small samples and is not easily calculated.

Recently, a new test statistic based on moments has been developed, g^2 (Ware & Ferron, 1995; Althouse, 1997; Althouse, Ware, & Ferron, 1997). This test statistic is modeled after K^2, but makes no attempt to normalize the measures of skewness and kurtosis nor does it rely on distributional theory to derive critical values. Rather, the distribution of g^2 was determined through intensive computer simulation. Findings from those studies have suggested that g^2 is competitive with both K^2 and an approximation to *W* for leptokurtic distributions, especially, and

does not suffer from the inflated Type I error rate evidenced by both K^2 and the approximate W (Ware & Ferron, 1995; Althouse, 1997; Althouse, Ware, & Ferron, 1997).

The current study was designed to derive the distribution of a test statistic based on normal probability plots. The first purpose was to provide an empirical derivation of the critical values for the Line Test (*LT*) with an extensive computer simulation. The second purpose was to determine the power of *LT* and compare it to the power of several other test statistics: g^2, K^2, the standardized third moment test ($\sqrt{b_1}$), the standardized fourth moment test (b_2), and a large sample approximation of the W test (Royston, 1992).

Logic of Tests Based on Normal Probability Plots

A normal probability plot is a plot of ordered observed data values as a function of expected values based on the assumption of normality. The various tests have all attempted to summarize the information available in these plots, in one way or another. The Shapiro-Wilk W test is focused on the slope, while the work of Filliben (1975), Weisberg and Bingham (1975), Looney and Gulledge (1985), and Verrill and Johnson (1988) is directed toward the correlation coefficient. If the data were "perfectly" normally distributed, one would expect the correlation coefficient to be equal to 1.0. As the data depart from normality, the expected value of r would be expected to decrease. To demonstrate, four variables were randomly generated for 200 cases: normal, uniform, $\chi^2(2)$, and $t(4)$. These variables represent normal, symmetric/flat, skewed/kurtotic, and symmetric/kurtotic distributions, respectively. Normal probability plots for these four variables are shown in Figure 1. An inspection of Figure 1 shows quite clearly that only the probability plot for the normal variate appears to be linear; the others are obviously non-linear. Thus, a correlation between the ordered data and their expected cumulative frequencies under the assumption of normality provides a measure of the degree to which the data are normally distributed.

Method

All the data were simulated using the SAS RANNOR function within PROC IML (SAS, 1995). The first Monte Carlo simulation was used to generate the critical values of *LT* by randomly generating 500,000 replications from a normal distribution for sample sizes 10(1)100(25)1000(250)5000. For each replication within each sample size condition, the value of the correlation coefficient was calculated. The steps to compute *LT* are as follows.

1. Arrange the observed set of data in ascending order.

2. Determine the percentile rank for each value in the set of data. The percentile rank value is defined as (j-0.5)/n, where j is the order of the value in step 1.

3. Calculate the standard normal quantiles for each of the percentile rank values. This set of values is the expected set of values based on the assumption of normality and can be obtained by using a normal probability table or by using an inverse normal distribution function.

4. Compute the correlation between the ordered set of data (Step 1) and the expected set of values (Step 3) to get *LT*.

An example of the computation of *LT* is provided in Table 1. The SPSS syntax and SAS code for computing *LT* are provided in Appendices A and B, respectively.

Given that the observed correlation decreases from 1.00 as the data depart from normality, the empirical critical values of *LT* for each sample size condition were obtained at the .10, .05, and .01 levels of significance by determining the 10^{th}, 5^{th}, and 1^{st} centiles of the 500,000 correlation coefficients at each sample size, respectively. Using plots of these estimated critical values versus sample size as a guide, the model of best fit was determined for each significance level using non-linear regression. These models served as the empirical distribution functions for *LT* and were used to derive the smoothed critical values for sample sizes between 10 and 5000.

The obtained estimated critical values were validated by conducting a smaller Monte Carlo simulation in which 10,000 samples were generated from a normal population for 45 samples sizes between 10 and 1000. The test statistic *LT* was calculated for each sample, as were the values of the other statistics (g^2, K^2, $\sqrt{b_1}$, b_2, and *W*). For each significance level, the proportion of times that each test statistic detected "non-normality" was determined.

To assess the power of *LT* relative to the power of the other five test statistics (g^2, K^2, $\sqrt{b_1}$, b_2, and *W*), another Monte Carlo simulation was conducted. Twelve alternative distributions were selected to ensure equal representation from the following six shapes of distributions: near normal (discrete), near normal (continuous), symmetric/flat, symmetric/peaked, skewed/flat and skewed/peaked. The 12 distributions are described in Table 2. To calculate power, 10,000 samples were generated for sample of sizes n = 10, 25, 50, 75, 100, 250, 300, 400, 500, 750, and 1000 for the twelve alternative distributions. The number of times each test statistic detected "non-normality" was determined.

Results

Calibration of *LT*

Empirical critical values were obtained for *LT* at the .10, .05, and .01 levels of significance. Examination of these values as a function of the 143 sample sizes indicated a nonlinear relationship (See Figure 2). Regression equations using inverse, inverse square roots, inverse cube roots, and inverse exponential functions were used to account for over 99% of the variance at each significance level. The three resulting regression equations were as follows:

$$Y'_{.10} = 0.996357 - 0.036317\left(\frac{1}{n}\right) + 0.003680\left(\frac{1}{\exp(n/200)}\right) - 0.432204\left(\frac{1}{\sqrt{n}}\right) + 0.160390\left(\frac{1}{\sqrt[3]{n}}\right).$$

$$Y'_{.05} = 0.995539 - 0.070244\left(\frac{1}{n}\right) + 0.004649\left(\frac{1}{\exp(n/200)}\right) - 0.522094\left(\frac{1}{\sqrt{n}}\right) + 0.195069\left(\frac{1}{\sqrt[3]{n}}\right)$$

$$Y'_{.01} = 0.993098 - 0.102328\left(\frac{1}{n}\right) + 0.007911\left(\frac{1}{\exp(n/200)}\right) - 0.786898\left(\frac{1}{\sqrt{n}}\right) + 0.298123\left(\frac{1}{\sqrt[3]{n}}\right)$$

The proportions of variance (rounded to four decimal places) explained were 1.0000, 1.0000, and .9999, respectively. Using these equations, the "smoothed" critical values of *LT* at each significance level were determined. Selected values are presented in Table 3.

<u>Type I Error Rates</u>

The "smoothed" values were used to validate *LT*. The results are presented in Table 4. The proportions for *LT* were similar to the theoretical proportions for each significance level, validating the use of the regression equations and indicating no problems with Type I error rates. Similarly, the Type I error rates for g^2 ,$\sqrt{b_1}$ and b_2 were fairly close to their expected values. However, for K^2, there was a slight, yet consistent, inflation of Type I error rates at the .05 and .01 levels of significance, particularly for small sample sizes. Even more troublesome were the Type I error rates for the approximation to *W*, which were largely inflated for all significance levels. As the sample size increased, these Type I error rates were more exaggerated. Overall, the approximation to *W* was the most liberal of the six tests. Once the Type I error rates were examined, the next step was to evaluate the power of *LT* to determine how well it could detect departures from normality and to see how its power compared to those of the competing test statistics. However, given the inflated Type I error rates for K^2 and W, we entered this next phase

suspecting that the power of K^2 and W would be somewhat inflated.

Power Results

Power values for the six test statistics were established for each of the 13 sample sizes for each of the twelve alternative distributions at each level of significance. We were also interested in seeing how these six test statistics differed across the various distribution categories and across different sample sizes. Therefore, we report the power results by distribution category. In addition, a visual examination of the power results indicated that there was an interaction between the type of test statistic, the distribution category of the alternative distributions, and the sample size. A full listing of the detailed power results at each of the significance levels for the 12 alternative distributions can be obtained by contacting the authors.

Near Normal Discrete Distributions. The near normal discrete distributions consisted of the binomial (20, .5) distribution and the Poisson (10) distribution. Each of these distributions have a skewness value close to zero and a kurtosis value close to 3, making them almost indistinguishable from the normal distribution, particularly for practical purposes such as the violation of the normality assumptions. The mean power of each test statistic for each sample size is provided in Table 5. The test statistic *LT* had relatively high power, trailing only behind the approximation of *W*. However, it was shown earlier that *W* had an inflated Type I error rate which may result into misleading higher power values. Regardless, for these near normal discrete distributions, *LT* and *W* were clearly the two most powerful tests.

Near Normal Continuous Distributions. The near normal continuous distributions consisted of the Tukey (1,5) distribution and the Johnson S Bounded (1,2) distribution. As with the earlier discrete distributions, the skewness value is close to zero and the kurtosis value is close to 3, making these distributions similar to the near normal discrete distribution, virtually indistinguishable from the normal distribution. The mean power of each test statistic for each sample size is given in Table 6. As with the discrete distributions, the test statistic *LT* performed nearly as well as the approximation to W, particularly for large sample sizes.

Symmetric/Platykurtic Distributions. This set of distributions included the Johnson B Bounded (0, .5) and the Tukey (1, 1.5) distributions. The mean powers for this distribution are provided in Table 7. The test statistic *LT* was as powerful as K^2, b_2, and the *W* approximation for sample sizes larger than 100 and only slightly less powerful for smaller sample sizes. The test statistic, b_2, had the highest power, but only slightly edged *W*, K^2, and *LT*. As

expected, the skewness test was the weakest test. One notable difference for this category of distributions from the near normal distributions was the rate at which the power increased, particularly for the smaller sample sizes. Therefore, sample sizes do not need to be large for most of the test statistics to have adequate power if we have data that are symmetric and flat.

Symmetric/Leptokurtic Distributions. The symmetric and peaked set consisted of Johnson S Unbounded (0, 2) and the Johnson S Unbounded (0,.9) distributions. The mean powers for this set of distributions are presented in Table 8. The test statistics g^2 and *LT* were as powerful or more powerful than all other test statistics for all sample sizes with g^2 slightly more powerful than *LT*. In fact, the powers of g^2, *LT*, K^2, b_2, and the *W* approximation were all high. Also, the mean powers for g^2 were notably higher from what they were in the previous sets of distributions. This pattern was also true for K^2, b_2, and $\sqrt{b_1}$. However, the *W* approximation and *LT* were slightly more powerful in detecting departures from normality for the near normal continuous distributions than this category. As with the symmetric/platykurtic distributions, $\sqrt{b_1}$ provided the weakest power.

Skewed/Platykurtic Distributions. The skewed and platykurtic distributions were represented by the Johnson S Bounded (.533, .5) and Beta (2,1) distributions. The mean powers for this set of distributions are presented in Table 9. As with the earlier flat shaped distributions, the *W* approximation was clearly the most powerful, particularly for sample sizes up to 100. For larger sample sizes, g^2, *LT*, K^2, $\sqrt{b_1}$ and the *W* approximation had equivalent power. The test statistic *LT* was second to the *W* approximation in power for smaller sample sizes, but became equal in power for sample sizes greater than 75. The patterns for g^2 and *LT* were similar to the powers for the symmetric, flat distributions indicating that the skewness of the distribution did not have an effect on power when the distribution was flat.

Skewed/Peaked Distributions. The distributions in this category included the Johnson S Unbounded (1,1) and the lognormal (0, 1, 0) distributions which are positively and negatively skewed, respectively. In addition, both these distributions have a kurtosis value larger than 3. The mean powers for this set are presented in Table 10. For sample sizes approaching 50, g^2 seemed nearly equivalent to *LT*, K^2, $\sqrt{b_1}$, and the *W* approximation. The power for *LT* was equivalent to the *W* approximation for all sample sizes. Also, *LT* was more powerful than all other tests, except the approximation to *W* for small sample sizes. Once again, for the skewed distributions, b_2 had the lowest power up until sample size 100 where it became equivalent in power to the other five statistics. All of the test

statistics demonstrated higher power for skewed/peaked distributions than for the other distribution categories considered earlier. In fact, absolute power was demonstrated at sample sizes greater than 100 for all test statistics.

Summary of the Results

Past research on using summary statistics (e.g., the correlation coefficient) to test for normality has shown that this family of statistics is conceptually easy and that the power compares favorably with leading test statistics (Filliben, 1975; Weisberg and Bingham, 1975; Looney and Gulledge, 1985; Verrill and Johnson, 1988). The results from this study support these conjectures. The Type I error rates *LT* were as expected for each significance level. Once again, the K^2 test statistic did not validate, having inflated Type I error rates at the .05 and .01 levels, and being slightly conservative at the .10 level. In addition, Royston's approximation of *W* had inflated Type I error rates at each significance level, indicating that it may be liberal in detecting departures from normality which may account for its high power. This inflated power is best seen when the power of the approximation to *W* for near normal distributions is considered. For large sample sizes, the *W* approximation had absolute power, yet for large sample sizes these distributions approximate the normal distribution. Therefore, low power in these cases is acceptable, perhaps even desirable. The two moment tests, $\sqrt{b_1}$, and b_2, while having high power for some distributions, do not provide an omnibus test for normality. That is, they only do well when the departure is due to skewness or kurtosis, respectively.

Overall, the *W* approximation appeared to be the most powerful test. However, the power of *W* could be biased due to its inflated Type I error rates. The test statistic, *LT*, was close in power to *W* for small sample sizes, and equivalent to *W* for large sample sizes and did not have an inflated Type I error rate. The test statistic, g^2, was the best overall test for peaked distributions regardless of sample size and symmetry. In particular, for symmetric, peaked distributions, the power of g^2 equaled or surpassed the other tests. The above results indicate that g^2 and *LT* are competitive with competing test statistics for many alternative distributions. In addition, this power is accomplished without a compromise in the Type I error rate.

Conclusions

As earlier mentioned, statistical packages differ in the test(s) they have implemented. Also, many of the tests for normality are not easy to implement, making them hard to use for the average researcher. The ideal test for

normality would demonstrate high power without an increased risk of a Type I error and would be easy to compute from the output of any of the major statistical packages. The results of this study indicate that *LT* has merit as a test for normality as it is unbiased, easy to compute, readily available, and powerful for a wide variety of situations.

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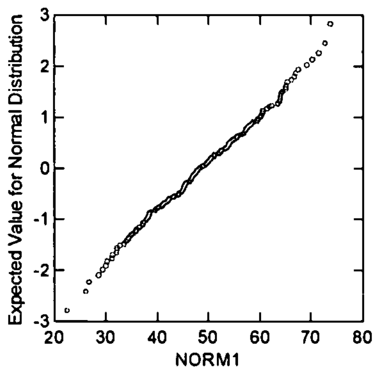
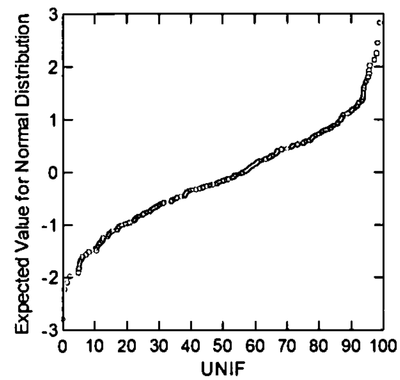
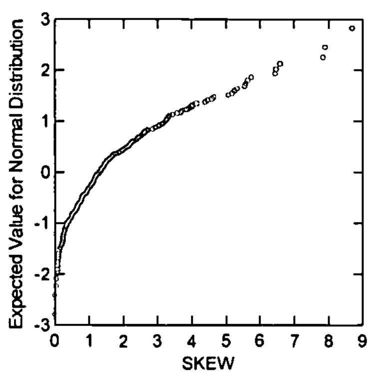
Figure 1. Normal probability plots for four different random variables: normal, uniform, skewed/kurtotic, and symmetric/kurtotic.



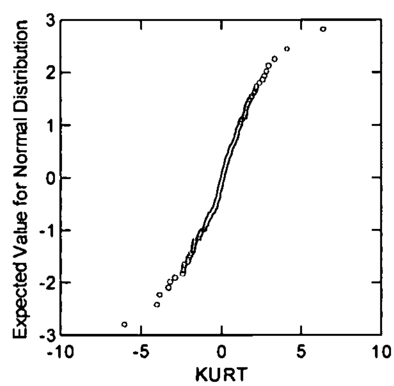
Normal



Uniform



Skewed/Kurtotic



Symmetric/Kurtotic

Figure 2. Plot of empirically derived critical values of *LT* for the .10, .05, and .01 levels as a function of sample size up to n = 100, inclusive.

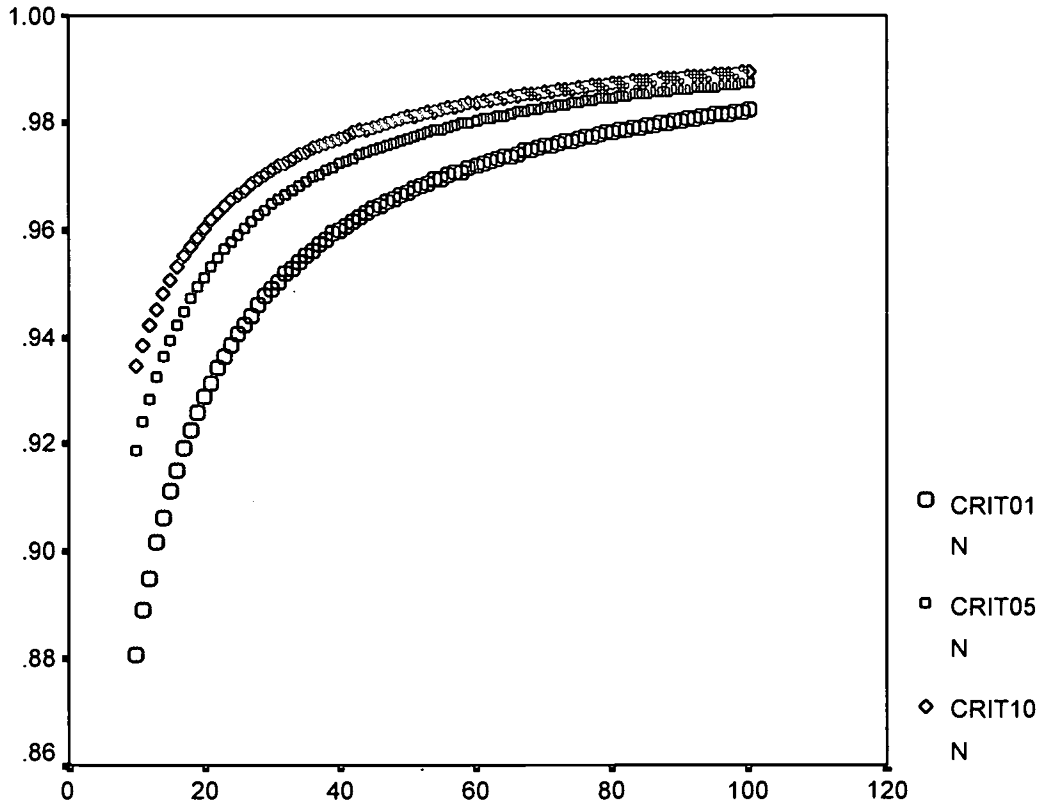


Table 1. Example for Computing *LT* for Sample of Size n = 10.

Sample Data Set	Step 1: Order Observations	Step 2: Determine Probability Values	Step 3: Calculate Standard Normal Quantiles	Step 4: Compute the Correlation
-.098	-.603	(1-.5)/10 = .05	-1.645	
.866	-.296	(2-.5)/10 = .15	-1.036	
-.77	-.277	(3-.5)/10 = .25	-.674	
-.603	-.134	(4-.5)/10 = .35	-.385	
-.296	-.098	(5-.5)/10 = .45	-.125	LT=.9274
.954	.011	(6-.5)/10 = .55	.125	
1.702	.024	(7-.5)/10 = .65	.385	
.011	.866	(8-.5)/10 = .75	.674	
.024	.954	(9-.5)/10 = .85	1.036	
-.134	1.702	(10-.5)/10 = .95	1.645	

Table 2. Categorization of the Alternative Distributions for the Power Study.

Distribution Category	Alternative Distributions (parameters)	$\sqrt{\beta_1}$	$\acute{\upsilon}_2$
Near Normal Discrete	Binomial (20, .5)	0	2.90
	Poisson (10)	.32	3.10
Near Normal Continuous	Tukey (1, 5)	0	2.90
	Johnson Bounded (1, 2)	.28	2.77
Symmetric/platykurtic	Johnson Bounded (0, .5)	0	1.63
	Tukey (1, 1.5)	0	1.75
Symmetric/leptokurtic	Johnson S Unbounded (0, 2)	0	4.71
	Johnson S Unbounded (0, .9)	0	82.08
Skewed/platykurtic	Johnson S Bounded (.533, .5)	.65	2.13
	Beta (2, 1)	-.57	2.40
Skewed/leptokurtic	Johnson S Unbounded (1, 1)	-5.30	93.40
	Lognormal (0, 1, 0)	6.18	113.94

Table 3. Critical Values of *LT* for a = .10, .05, & .01 as a function of sample size.

Sample Size	.10	.05	.01
10	0.93400	0.91838	0.87993
11	0.93834	0.92385	0.88808
12	0.94209	0.92855	0.89508
13	0.94535	0.93265	0.90119
14	0.94823	0.93626	0.90656
15	0.95079	0.93946	0.91133
16	0.95308	0.94233	0.91559
17	0.95515	0.94492	0.91944
18	0.95703	0.94726	0.92293
19	0.95874	0.94940	0.92611
20	0.96032	0.95135	0.92902
22	0.96310	0.95482	0.93416
24	0.96549	0.95779	0.93858
26	0.96757	0.96037	0.94242
28	0.96940	0.96264	0.94579
30	0.97102	0.96466	0.94878
32	0.97247	0.96645	0.95144
34	0.97378	0.96807	0.95384
36	0.97496	0.96953	0.95600
38	0.97604	0.97086	0.95797
40	0.97702	0.97208	0.95977
45	0.97915	0.97470	0.96366
50	0.98091	0.97687	0.96685
55	0.98239	0.97869	0.96954
60	0.98365	0.98024	0.97182
65	0.98474	0.98158	0.97379
70	0.98569	0.98274	0.97550
80	0.98727	0.98468	0.97833
90	0.98853	0.98622	0.98059
100	0.98956	0.98747	0.98241
125	0.99146	0.98978	0.98576
150	0.99275	0.99135	0.98801
175	0.99369	0.99248	0.98963
200	0.99439	0.99334	0.99083
300	0.99606	0.99534	0.99363
400	0.99692	0.99636	0.99503
500	0.99747	0.99701	0.99591
750	0.99827	0.99796	0.99723
1000	0.99872	0.99850	0.99798
1500	0.99919	0.99906	0.99876
2000	0.99940	0.99931	0.99911
2500	0.99952	0.99944	0.99929
3000	0.99957	0.99951	0.99937
3500	0.99960	0.99954	0.99940
4000	0.99962	0.99955	0.99941
∞	1.00000	1.00000	1.00000

Table 4. Comparison of Type I Error Rates for Six Competing Statistics.

Level of Significance	n	g^2	K^2	$\sqrt{b_1}$	b_2	*LT*	*W app.*
α=.10	10	.105	.096	.102	.090	.101	.098
	25	.103	.099	.102	.100	.101	.119
	50	.097	.098	.101	.102	.100	.124
	75	.103	.103	.104	.102	.102	.125
	100	.100	.097	.099	.104	.100	.129
	150	.097	.096	.097	.105	.097	.132
	200	.102	.102	.101	.102	.100	.141
	250	.099	.097	.099	.099	.101	.144
	300	.098	.099	.099	.098	.100	.140
	400	.099	.096	.099	.103	.099	.153
	500	.099	.102	.103	.109	.099	.157
	750	.101	.102	.099	.105	.100	.160
	1000	.099	.100	.100	.097	.101	.170
Mean Type I Error Rate at α=.10		.1002	.0990	.1004	.1012	.1001	.1378
α=.05	10	.051	.061	.054	.044	.052	.053
	25	.050	.060	.052	.053	.051	.063
	50	.049	.057	.051	.054	.047	.066
	75	.052	.059	.052	.056	.056	.073
	100	.049	.054	.047	.055	.050	.072
	150	.049	.054	.047	.055	.047	.074
	200	.053	.055	.051	.054	.051	.084
	250	.049	.051	.047	.051	.049	.084
	300	.048	.052	.048	.050	.050	.087
	400	.050	.051	.049	.052	.052	.093
	500	.051	.053	.049	.056	.049	.097
	750	.051	.053	.051	.054	.052	.104
	1000	.050	.051	.053	.051	.051	.113
Mean Type I Error Rate at α=.05		.0502	.0547	.0501	.0527	.0505	.0818
α=.01	10	.012	.025	.013	.007	.103	.013
	25	.011	.022	.011	.011	.012	.016
	50	.009	.018	.009	.011	.008	.014
	75	.011	.019	.011	.014	.011	.021
	100	.012	.019	.010	.015	.010	.020
	150	.009	.016	.010	.012	.010	.022
	200	.010	.014	.010	.012	.010	.025
	250	.011	.014	.010	.012	.010	.025
	300	.010	.013	.008	.013	.009	.027
	400	.010	.014	.010	.012	.011	.034
	500	.009	.014	.011	.012	.009	.032
	750	.013	.014	.011	.012	.011	.040
	1000	.011	.013	.010	.012	.011	.045
Mean Type I Error Rate at α=.01		.0116	.0165	.0103	.0119	.0104	.0257

Table 5. Mean Power Values for Near Normal Discrete Distributions[a]: By alpha, sample size, and test statistic.

		Test of Normality					
Alpha	N	g^2	K^2	Skew	Kurtosis	*W* Approx	*LT*
	10	10.0	9.3	10.4	8.8	16.2	15.9
	25	10.8	10.6	11.6	10.3	27.0	22.2
	50	13.0	13.4	14.7	11.7	46.9	38.4
	75	14.7	14.9	17.8	11.8	68.1	58.8
	100	16.9	17.4	20.7	12.4	84.4	77.2
.10	150	21.6	21.8	26.2	12.4	96.8	93.8
	200	25.4	26.2	30.7	13.2	99.8	99.1
	250	29.8	30.4	35.4	13.2	100.	100.
	300	34.0	34.6	39.0	14.1	100.	100.
	400	40.0	40.9	44.5	14.2	100.	100.
	500	44.8	45.8	48.3	14.8	100.	100.
	750	51.5	52.8	52.4	17.1	100.	100.
	1000	54.1	55.4	53.6	18.4	100.	100.
	10	5.0	5.8	5.2	4.0	8.8	8.0
	25	5.4	6.5	6.1	5.4	15.6	11.6
	50	7.0	8.3	8.4	6.6	30.2	21.2
	75	8.0	9.6	10.7	6.5	49.9	36.3
	100	9.4	11.0	13.0	6.8	70.4	55.0
.05	150	12.6	14.3	18.1	7.2	91.6	84.0
	200	15.9	17.8	22.0	7.6	98.6	94.8
	250	1934	21.3	27.0	7.8	99.9	99.2
	300	23.6	25.4	30.8	8.2	100.	100.
	400	30.4	32.0	37.4	8.4	100.	100.
	500	36.2	38.0	42.2	8.8	100.	100.
	750	48.5	47.3	48.7	10.7	100.	100.
	1000	50.1	51.6	50.9	11.5	100.	100.
	10	.8	2.3	1.0	.5	2.0	1.6
	25	1.3	2.6	1.5	1.2	4.0	2.6
	50	1.8	3.2	2.4	1.6	9.4	4.8
	75	2.0	4.0	3.4	1.8	18.8	9.3
	100	2.6	4.7	4.5	2.0	34.0	15.6
.01	150	3.7	6.4	7.2	2.3	70.6	41.6
	200	4.9	8.2	10.1	2.4	89.3	72.4
	250	6.4	10.1	13.2	2.3	97.4	87.5
	300	8.8	13.2	17.0	2.7	99.7	95.0
	400	13.2	18.6	23.8	2.8	100.	99.9
	500	17.8	23.8	29.8	3.2	100.	100.
	750	32.0	35.8	40.8	3.8	100.	100.
	1000	41.6	44.4	46.7	4.3	100.	100.

[a] Binomial (20, 0.5) Skew = 0.00 Kurtosis = 2.90
Poisson (10) Skew = 0.32 Kurtosis = 3.10

Table 6. Mean Power Values for Near Normal Continuous Distributions[b]: By alpha, sample size, and test
statistic.

Alpha	N	g^2	K^2	Skew	Kurtosis	*W* Approx	*LT*
		Test of Normality					
.10	10	11.0	9.7	10.6	9.4	11.3	11.9
	25	8.2	7.6	8.4	8.2	15.6	13.3
	50	7.4	8.3	9.4	8.4	25.2	18.4
	75	8.6	10.0	12.4	8.4	37.8	27.0
	100	10.8	12.5	14.8	9.0	50.6	36.8
	150	16.4	18.2	20.6	10.0	69.6	56.6
	200	22.8	24.7	26.2	11.3	80.8	70.4
	250	28.8	30.8	30.6	12.4	86.8	79.6
	300	34.0	35.6	34.3	13.4	91.0	85.0
	400	41.8	43.0	40.5	15.8	96.0	92.1
	500	46.5	47.2	45.0	17.8	98.4	96.3
	750	49.9	50.4	49.2	22.6	99.8	99.6
	1000	50.6	51.1	50.3	28.0	100.	99.9
.05	10	5.4	6.1	5.4	4.4	6.0	6.4
	25	3.7	4.2	4.0	3.8	8.4	6.7
	50	3.0	4.6	4.9	4.3	14.9	9.4
	75	3.3	5.4	6.4	4.6	24.6	15.0
	100	4.0	6.8	8.5	5.0	36.8	22.2
	150	6.5	10.3	12.8	6.1	58.6	40.2
	200	10.4	15.2	17.6	6.8	72.8	57.1
	250	15.7	20.6	22.0	7.8	81.2	69.8
	300	21.2	26.0	26.0	8.4	86.4	77.3
	400	31.6	35.4	33.7	10.2	93.4	86.3
	500	40.1	42.5	39.7	11.8	97.2	92.9
	750	48.4	49.0	47.0	15.6	99.7	98.8
	1000	50.0	50.3	49.4	20.3	100.	99.9
.01	10	1.1	2.5	1.2	.6	1.4	1.4
	25	.6	1.3	.8	.6	2.1	1.4
	50	.4	1.3	.9	1.1	3.9	1.8
	75	.4	1.5	1.2	1.2	8.2	3.0
	100	.4	1.9	1.9	1.6	14.6	4.7
	150	.6	3.0	3.7	1.9	33.2	13.1
	200	1.0	4.2	6.0	2.4	52.6	26.0
	250	1.4	6.2	8.7	2.6	66.6	42.4
	300	2.6	9.4	11.8	3.0	74.8	54.8
	400	6.1	17.1	18.4	3.8	85.2	71.4
	500	13.0	25.6	25.5	4.6	92.5	80.9
	750	35.0	42.2	38.6	6.8	98.8	94.2
	1000	46.2	48.5	47.6	9.8	99.9	99.0

[b] Tukey (1,5) Skew = 0.00 Kurtosis = 2.90
Johnson S Bounded (1, 2) Skew = 0.28 Kurtosis = 2.77

Table 7. Mean Power Values for Symmetric/Platykurtic Continuous Distributions[c]: By alpha, sample size, and test statistic.

Alpha	N	g^2	K^2	Skew	Kurtosis	*W* Approx	*LT*
		Test of Normality					
	10	3.4	9.4	5.4	20.2	24.9	18.2
	25	1.8	56.2	2.2	70.6	69.2	49.8
	50	55.6	95.6	1.4	97.4	97.2	90.8
	75	93.2	99.8	1.2	99.8	99.9	99.2
	100	99.4	100.	.8	100.	100.	99.9
.10	150	100.	100.	.9	100.	100.	100.
	200	100.	100.	.8	100.	100.	100.
	250	100.	100.	.8	100.	100.	100.
	300	100.	100.	.8	100.	100.	100.
	400	100.	100.	.8	100.	100.	100.
	500	100.	100.	.8	100.	100.	100.
	750	100.	100.	.6	100.	100.	100.
	1000	100.	100.	.7	100.	100.	100.
	10	1.4	3.6	2.3	10.4	12.9	8.4
	25	.1	41.8	.6	59.0	52.6	31.2
	50	.4	91.4	.3	95.1	93.4	80.4
	75	35.2	99.2	.3	99.5	99.5	97.3
	100	83.5	99.9	.2	100.	100.	99.7
.05	150	99.8	100	.2	100.	100.	100
	200	100	100	.2	100	100	100
	250	100	100	.2	100	100	100
	300	100	100	.2	100	100	100
	400	100	100	.2	100	100	100
	500	100	100	.2	100	100	100
	750	100	100	.2	100	100	100
	1000	100	100	.2	100	100	100
	10	.3	.7	.4	1.0	2.2	1.1
	25	.0	20.4	.0	34.0	20.2	6.0
	50	.0	79.2	.0	86.4	74.6	44.0
	75	.5	96.8	.0	98.2	96.2	82.9
	100	.0	99.6	.0	99.8	99.6	96.5
.01	150	8.9	100.	.0	100.	100.	99.9
	200	79.6	100.	.0	100.	100.	100.
	250	99.3	100.	.0	100.	100.	100.
	300	100.	100.	.0	100.	100.	100.
	400	100.	100.	.0	100.	100.	100.
	500	100.	100.	.0	100.	100.	100.
	750	100.	100.	.0	100.	100.	100.
	1000	100.	100.	.0	100.	100.	100.

[c] Johnson S Bounded (0, 0.5) Skew = 0.0 Kurtosis = 1.63
Tukey (1, 1.5) Skew = 0.0 Kurtosis = 1.75

Table 8. Mean Power Values for Symmetric/Leptokurtic Continuous Distributions[d]: By alpha,

sample size, and test statistic.

Alpha	N	g^2	K^2	Skew	Kurtosis	*W* Approx	*LT*
		Test of Normality					
	10	29.2	27.0	27.6	23.6	24.7	27.2
	25	49.8	45.4	41.4	43.1	44.8	46.9
	50	64.5	60.2	49.2	59.6	60.8	62.4
	75	72.2	68.0	53.6	67.9	68.0	69.3
	100	76.3	72.4	56.0	73.0	72.8	73.6
.10	150	82.0	78.6	58.9	79.4	79.0	79.3
	200	86.2	82.9	60.8	84.4	83.4	83.4
	250	90.2	87.1	62.6	88.9	87.8	87.8
	300	91.8	89.4	63.0	91.2	90.2	90.0
	400	95.4	93.6	64.6	95.4	94.2	93.8
	500	97.4	96.3	65.2	97.4	96.8	96.4
	750	99.4	99.2	67.8	99.6	99.4	99.2
	1000	99.9	99.8	68.8	99.9	99.9	99.8
	10	20.4	21.7	20.4	16.7	18.2	19.8
	25	40.7	39.2	33.6	35.2	37.7	39.0
	50	57.6	54.6	42.4	52.6	55.2	55.7
	75	66.2	62.8	47.0	62.0	63.6	64.0
	100	71.0	67.6	50.0	67.6	68.8	68.6
.05	150	77.6	74.2	53.6	74.6	75.0	74.6
	200	82.2	78.6	55.1	79.6	80.0	79.0
	250	86.6	83.0	56.9	84.6	84.4	83.5
	300	89.2	85.8	57.7	87.5	87.5	86.2
	400	93.4	90.8	59.6	92.8	92.3	91.0
	500	96.0	94.2	60.0	95.8	95.6	94.5
	750	99.0	98.5	62.9	99.0	99.0	98.6
	1000	99.8	99.7	64.2	99.8	99.8	99.7
	10	9.8	14.2	10.1	7.3	9.4	9.7
	25	26.0	29.2	22.0	22.2	27.0	26.8
	50	43.8	44.4	31.8	40.0	46.5	44.6
	75	54.5	53.7	36.8	51.0	55.8	54.5
	100	60.6	59.4	40.2	57.7	61.4	59.8
.01	150	67.6	66.6	44.4	65.2	68.0	65.9
	200	72.6	70.4	46.0	70.1	73.2	70.2
	250	77.1	74.6	48.2	74.8	78.0	74.7
	300	80.6	77.6	49.1	78.5	81.2	77.6
	400	86.6	83.8	51.2	85.4	87.8	84.0
	500	91.2	88.4	52.2	90.5	92.2	88.8
	750	97.4	95.8	54.6	97.0	97.9	96.0
	1000	99.2	98.8	56.3	99.2	99.5	98.7

[d]Johnson S Unbounded (0,2) Skew = 0.0 Kurtosis = 4.71
Johnson S Unbounded (0, 0.9) Skew = 0.0 Kurtosis = 82.08

Table 9. Mean Power Values for Skewed/Platykurtic Continuous Distributions[e]: By alpha, sample size, and test statistic.

		Test of Normality					
Alpha	N	g^2	K^2	Skew	Kurtosis	*W* Approx	*LT*
	10	12.9	15.4	18.2	16.9	35.6	30.9
	25	22.3	36.2	32.8	29.4	77.8	68.0
	50	71.5	78.6	52.9	43.0	97.3	94.0
	75	93.0	95.1	78.7	53.6	99.8	99.4
	100	98.8	99.2	89.6	62.8	100.	99.9
.10	150	100.	100.	98.0	74.7	100.	100.
	200	100.	100.	99.8	83.4	100.	100.
	250	100.	100.	99.9	88.8	100.	100.
	300	100.	100.	100.	92.8	100.	100.
	400	100.	100.	100.	96.9	100.	100.
	500	100.	100.	100.	98.6	100.	100.
	750	100.	100.	100.	99.8	100.	100.
	1000	100.	100.	100.	100.	100.	100.
	10	6.6	9.3	10.6	9.2	22.9	18.9
	25	7.2	21.8	18.8	20.8	66.5	54.0
	50	20.2	58.6	39.6	34.8	94.2	87.4
	75	62.4	84.4	61.6	45.4	99.5	97.8
	100	85.5	96.0	77.5	54.4	100.	99.8
.05	150	99.4	99.9	94.2	67.7	100.	100.
	200	100.	100.	98.8	77.4	100.	100.
	250	100.	100.	99.8	84.0	100.	100.
	300	100.	100.	100.	88.8	100.	100.
	400	100.	100.	100.	94.5	100.	100.
	500	100.	100.	100.	97.3	100.	100.
	750	100.	100.	100.	99.6	100.	100.
	1000	100.	100.	100.	99.9	100.	100.
	10	2.1	4.0	2.8	1.7	7.4	5.2
	25	.9	7.8	4.0	9.0	40.2	25.1
	50	.6	24.0	10.8	21.6	81.6	58.4
	75	1.4	58.2	23.9	30.8	96.3	88.4
	100	4.6	71.8	40.3	39.4	99.6	97.0
.01	150	49.6	96.4	71.8	53.1	100.	99.5
	200	83.5	99.8	90.0	64.0	100.	100.
	250	98.5	100.	96.9	72.2	100.	100.
	300	99.9	100.	99.2	78.8	100.	100.
	400	100.	100.	100.	87.2	100.	100.
	500	100.	100.	100.	92.7	100.	100.
	750	100.	100.	100.	98.4	100.	100.
	1000	100.	100.	100.	99.7	100.	100.

[e] Johnson S Bounded (0.533, 0.5) Skew = 0.65 Kurtosis = 2.13
Beta (2,1) Skew = -0.57 Kurtosis = 2.40

Table 10. Mean Power Values for Skewed/Leptokurtic Continuous Distributions[f]: By alpha, sample size, and test statistic.

		Test of Normality					
Alpha	N	g^2	K^2	Skew	Kurtosis	*W* Approx	*LT*
	10	51.4	50.4	57.0	39.4	59.8	59.8
	25	89.0	87.0	91.2	71.6	92.9	92.5
	50	99.2	98.8	98.9	91.5	99.4	99.4
	75	99.9	99.9	99.7	97.6	99.9	99.9
	100	100.	100.	99.9	99.4	100.	100.
.10	150	100.	100.	99.9	99.9	100.	100.
	200	100.	100.	100.	100.	100.	100.
	250	100.	100.	100.	100.	100.	100.
	300	100.	100.	100.	100.	100.	100.
	400	100.	100.	100.	100.	100.	100.
	500	100.	100.	100.	100.	100.	100.
	750	100.	100.	100.	100.	100.	100.
	1000	100.	100.	100.	100.	100.	100.
	10	39.8	43.6	46.8	32.2	50.5	50.2
	25	81.1	82.5	86.8	64.8	90.4	89.5
	50	97.8	97.9	98.2	88.0	99.1	99.0
	75	99.8	99.7	99.6	96.2	99.9	99.9
	100	99.8	99.9	99.8	99.0	100.	100.
.05	150	100.	100.	99.9	99.9	100.	100.
	200	100.	100.	100.	100.	100.	100.
	250	100.	100.	100.	100.	100.	100.
	300	100.	100.	100.	100.	100.	100.
	400	100.	100.	100.	100.	100.	100.
	500	100.	100.	100.	100.	100.	100.
	750	100.	100.	100.	100.	100.	100.
	1000	100.	100.	100.	100.	100.	100.
	10	24.5	32.4	28.0	19.5	33.2	31.6
	25	62.8	73.0	74.2	49.9	83.1	79.9
	50	91.0	95.1	95.8	78.5	98.0	97.2
	75	98.3	99.2	99.0	91.6	99.7	99.6
	100	99.8	99.8	99.6	97.1	100.	100.
.01	150	100.	100.	99.9	99.6	100.	100.
	200	100.	100.	99.9	99.9	100.	100.
	250	100.	100.	100.	100.	100.	100.
	300	100.	100.	100.	100.	100.	100.
	400	100.	100.	100.	100.	100.	100.
	500	100.	100.	100.	100.	100.	100.
	750	100.	100.	100.	100.	100.	100.
	1000	100.	100.	100.	100.	100.	100.

[f] Chi-squared (2) Skew = 5.30 Kurtosis = 93.40
Lognormal (0,1,0) Skew = 6.18 Kurtosis = 113.94

Appendix A

SPSS Syntax for Computing the Line Test (LT) for a Given Dataset and for Graphing the Normal Probability Plot

* Testing for univariate normality.

* Assume that the name of your variable is "score".

* First establish the number of cases by changing "**xxxx**" to the actual number of cases.

compute n = **xxxx**.

* Now rank the cases according to their values, substituting your variable name for **varname**.

rank variables = **varname** (A) / rank/ print=yes/ties=mean.

* Compute the percentile rank for each case, changing **rvarname** to rname.

* For example, if your variable name is "score", you would put "rscore" in the next line.

compute perc = (**rvarname** - .5)/n.

* Now translate the percentile rank to a normally distributed variate.

compute norm = idf.normal(perc,0,1).

execute.

* Graph the normal probability plot, changing **varname** to "score".

graph / scatterplot (bivar) = **varname** with norm.

* Now compute the value of *LT*.

correlations / variables = **varname** norm.

Note: You can run this syntax file by making the appropriate changes and deleting the comment lines (those lines beginning with "*". For example, suppose that you had 72 cases and the variable name was "ability.

compute n = **72**.

rank variables = **ability** (A) / rank/ print=yes/ties=mean.

compute perc = (**rability** - .5)/n.

compute norm = idf.normal(perc,0,1).

execute.

graph / scatterplot (bivar) = **ability** with norm.

correlations / variables = ability norm.

Appendix B

SAS Code Computing the Line Test (LT) for a Given Dataset

Note: This code assumes that any missing values for the variables you are testing have been removed.

```
proc iml;
use data_set_name;
read all var {variable_name} into p;
create dsl var {LT};

/* Step 1 - Arrange the observed data set in ascending order */
s=p;
s(|rank(s)|)=p;
p=s;

/* Step 2 - Determine the probability value for each value in the dataset */
n=nrow(p);
i=1:n;
problev=i-.5/n;

/* Step 3 - Calculate the standard normal quantiles for each probability value */
m=probit((i-.5)/n);
m=m`;

/* Step 4 - Compute LT */
psqr=p##2;
msqr=m##2;
pm=p#m;
psum=sum(p);
psqrsum=sum(psqr);
msum=sum(m);
msqrsum=sum(msqr);
pmsum=sum(pm);
ltnum=((n)*(pmsum))-((psum)*(msum));
ltden=sqrt(((n*psqrsum)-(psum*psum))*((n*msqrsum)-(msum*msum)));
LT=ltnum/ltden;
print LT;
append;
```

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